Job Sequencing

Sequencing is the selection of an appropriate order in which a number of jobs (operations) can be assigned to a finite number of service facilities (machines) so as to optimize the output in terms of time, cost or profit.

General Sequencing problem :- Let there be n different jobs (1, 2, ... n) each of which has to be processed, one at a time on each of m machines ($M_1, M_2, \ldots$). Each job is processed on machines in order like $M_1, M_2, \ldots\ldots$ i.e. passing is not allowed.

The sequencing problem is to find the sequence of jobs to be executed so that the total cost or time involved is minimize.

Assumptions of Sequencing problem :-

1) All jobs are completly known and are ready for processing.
2) Machines is to used are of different types.

3) Only one job can be processed on a given machine at a time.

4) The processing time on each machine are exactly known.

5) Processing times are independent of the order of job in which they are to be processed.

6) A job once started must be completed.

7) The time required to move jobs from one machine to another is negligible.

Terminology used in Sequencing :-

1) **Number of machine** – It refers to the number of service facilities through which a job must pass before it is assumed to be completed.

2) **Processing order** – The order in which the given machines are required for completing the jobs.

3) **Processing time** – It is the time required by a job on each machine.

4) Idle time on a machine — It is the time for which a machine does not have a job to process.

5) Total elapsed Time — It is the time interval b/w the start of first job and completion of the last job (including idle time) in a particular order by the given set of machines

6) No passing Rule — The rule by which we maintain the order of jobs are to be processed on the given set of machines.

Types of Sequencing problems (Johnson's Rule) :—

There are four different cases which are as follows —

1) n Jobs on Two machines
2) n Jobs on Three machines
3) Two jobs on m machines
4) n Jobs on m machines.

Type I : Processing n Jobs Through Two machines :-

Consider two machines A and B are involved and each job is to be processed on machine A and then on machine B. The processing time of jobs on each machines are given as—

	processing time on machine				
Jobs →	1	2	3	. . . . .	n
Machine A	t_{A1}	t_{A2}	t_{A3}	. . . .	t_{An}
Machine B	t_{B1}	t_{B2}	t_{B3}	. . . . .	t_{Bn}

The algorithm of solving the above problem is defined as—

1) Select the smallest processing time in each column i.e. find out minimum (t_{Ai} , t_{Bi}) for all i.

2) There are following cases—

(a) If the smallest processing time corresponds to machine A then assign the corresponding job as 1st job and if it corresponds to the machine B then assign the job as last job in the sequence.

(b) If there is a tie in selecting the minimum processing time then three situations may arise as follows –

(i) minimum among all processing times is same for the machine i.e. min $(t_{Aj}, t_{Bj}) = t_{AK} = t_{Br}$, then process the Kth job first and rth job last.

(ii) If tie occurs on machine M_1 only, then select the job corresponding to the smallest job subscript first.

(iii) If the tie occurs on machine M_2 only, then select the job corresponding to the largest job subscript last.

(3) Cross off the jobs already assigned and repeat all above step until all jobs have been assigned.

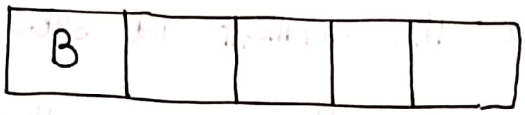
Example – Five jobs are performed first on machine M_1 then on machine M_2. Time in hours taken by each job on each machine is given below –

Machines ↓	Jobs → A	B	C	D	E
M_1	5	1	9	3	10
M_2	2	6	7	8	4

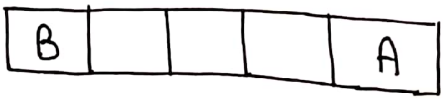
Determine the optimum sequnce of jobs and the minimum time elapsed.

Sol- It is the case of two machines and five jobs, and the order of machines is M_1, M_2.

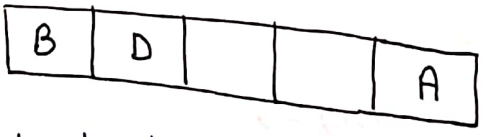
The smallest processing time b/w the two machine is 1 which corresponds to job B on machine M_1. So the job B will be processed first as-



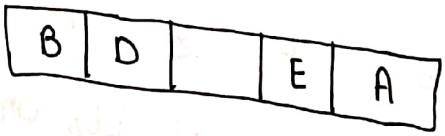
After the job B, the minimum processing time is 2 which corresponds to job A on machine M_2. Therefore, according to rule, job A will be processed in last.



Now, minimum processing time is 3 for job D on machine M_1, therefore assign job D at place 2 in the sequence as-



In the reduced problem, the minimum processing time is 4 which corresponds to job E on machine M_2. Hence job E assigned as-



Now, there is only one empty place in the sequence and one job C is left to be assigned, so job C is assigned in third place —

B	D	C	E	A

Now, calculation of minimum elapsed time can be done in tabular form as —

Sequence of jobs	Machine M_1			Machine M_2			
	Time in	processing Time	Time out	Time in	processing Time	Time out	Idle Time
B	0	1	0+1=1	1	6	1+6=7	1
D	1	3	1+3=4	7	8	7+8=15	—
C	4	9	4+9=13	15	7	15+7=22	—
E	13	10	13+10=23	23	4	23+4=27	1
A	23	5	23+5=28	28	2	28+2=30	1

So, minimum elapsed time = 30 − 0 = 30 hours

and idle time for machine M_1 = 30 − 28 = 2 hours

& idle time for machine M_2 = 1+1+1 = 3 hours

Example – six jobs are to be processed on two machines A and then on machine B. Time in hours taken by each job on each machine is given below –

Jobs →

	1	2	3	4	5	6
Machine A	5	3	2	10	12	6
Machine B	3	2	5	11	10	7

Determine the elapsed time.

Sol

Select minimum processing time between the two machine A and B and assigned the jobs one by one.

The resultant sequence plan of jobs is as follows

3	6	4	5	1	2

Now, calculation of minimum elapsed time is as –

Sequencing of Jobs	Machine A			Machine B			
	Time in	Processing Time	Time out	Time in	Processing Time	Time out	Idle Time
3	0	2	0+2=2	2	5	2+5=7	2
6	2	6	2+6=8	8	7	8+7=15	1
4	8	10	8+10=18	18	11	18+11=29	3
5	18	12	18+12=30	30	10	30+10=40	1
1	30	5	30+5=35	40	3	40+3=43	—
2	35	3	35+3=38	43	2	43+2=45	—

Example - In a factory, there are seven jobs to perform, each of which should go through two machine A and B. The processing time (in hrs.) for the jobs are as given below.

(i)

Machine ↓	Jobs						
	1	2	3	4	5	6	7
machine A	4	13	16	7	11	12	10
machine B	9	11	11	7	13	2	4

(ii)

Machine ↓	Jobs								
	A	B	C	D	E	F	G	H	I
Machine A	4	7	6	11	8	10	9	7	6
machine B	8	10	9	6	5	11	5	10	13

Type II : Processing n Jobs Through Three Machines :-

Suppose there are three machines A, B and C and the order of processing on these machines is A → B → C

There is no solution available for the general sequencing problems of n jobs through 3 machines. So we convert the given problem in 2 machines.

There are some conditions which are as -

1) The minimum time on machine A is greater than or equal to the maximum time on machine B.

i.e. $\text{Min } A_i \geq \text{Max } B_i$

2) The minimum time on machine C is greater than or equal to the maximum time on machine B.

i.e. $\text{Min } C_i \geq \text{Max } B_i$

If either one or both conditions are satisfied then we convert three machines problem in two machine as follows -

$$G_i = A_i + B_i$$

&

$$H_i = B_i + C_i \quad (i = 1, 2, 3, \ldots n)$$

} new processing times

G and H are equivalent machine of given machine.

Example - Find the sequence that minimizes the total elapsed

Example - Find the sequence that minimizes the total elapsed time (in hours) required to complete the following jobs on three machines M_1, M_2, M_3 in the order $M_1 \rightarrow M_2 \rightarrow M_3$.

Machines	Jobs				
	A	B	C	D	E
M_1	6	8	7	10	6
M_2	3	2	5	6	4
M_3	4	8	6	7	8

Sol - Here, $\min(M_1) = 6$, $\min(M_3) = 4$ & $\max(M_2) = 6$

Therefore,

$$\min(M_1) \geq \max(M_2) \quad \& \quad \min(M_3) \geq \max(M_2)$$

Condition satisfied.

Now, we convert the given problem in 2 machines A & B such that -

Jobs	$A = M_1 + M_2$	$B = M_2 + M_3$
A	9	7
B	10	10
C	12	11
D	16	13
E	10	12

Now, optimal sequence of jobs by using machines are as -

E	B	D	C	A

The minimum elapsed time can be calculated as -

Job Sequence	Machine M_1			Machine M_2				Machine M_3			
	Time in	Processing Time	Time out	Time in	Processing Time	Time out	Idle Time	Time in	Processing Time	Time out	Idle Time
E	0	6	6	6	4	10	6	10	8	18	10
B	6	8	14	14	2	16	4	18	8	26	—
D	14	10	24	24	6	30	8	30	7	37	4
C	24	7	31	31	5	36	1	37	6	43	—
A	31	6	37	37	3	40	1	43	4	47	—

So, minimum elapsed time = 47 hours

Idle time, for machine M_1 = 47 - 37 = 10 hours

for machine M_2 = 6+4+8+1+1 = 20 hours + 7 hours

for machine M_3 = 10+4 = 14 hours

Example - Find the sequence that minimizes the total time in hours required to complete the following tasks -

Machines	Tasks						
	A	B	C	D	E	F	G
m/c 1	3	8	7	4	9	8	7
m/c 2	4	3	2	5	1	4	3
m/c 3	6	7	5	11	5	6	12

What is the minimum elapsed time?

Type III : Processing n Jobs on K machines :—

Let jobs are 1, 2, 3, ... k and machines are A, B, C, ... K and the order of processing is $A \to B \to C \ldots\ldots \to K$.

The processing time of each ~~machine~~ jobs on each machines are as follows —

Jobs	A	B	C	.....	K
1	A_1	B_1	C_1	.....	K_1
2	A_2	B_2	C_2	.....	K_2
3	A_3	B_3	C_3	.....	K_3
⋮	⋮	⋮	⋮		⋮
n	A_n	B_n	C_n	.....	K_n

If either or both of conditions given below are satisfied with one can proceed –

Condition 1 : Min $A_i \geq$ Max of $B, C, \ldots K-1$

Condition 2 : Min $K \geq$ Max of $B, C, \ldots K-1$

One can replace 'K' machines by an equivalent two machine problem. These two machines are G and H where

$$G_i = A_i + B_i + C_i + \ldots\ldots (K-1)_i$$

&

$$H_i = B_i + C_i + \ldots\ldots K_i$$

Determine the optimal sequence for the the n jobs and 2 machines by following the usual procedure as described earlier.

Example – There are four jobs each of which has to be processed on machines A, B, C, D, E and F in the order ABCDEF. Processing time in hours is given below. Find out the optimal sequencing of jobs and elapsed time.

Jobs	Machines					
	A	B	C	D	E	F
1	15	8	6	14	6	26
2	17	7	9	10	15	22
3	21	7	12	9	11	19
4	18	6	11	12	14	17

Sol - Here, $\min(A_i) = 15$, $\max(B_i) = 8$, $\max(C_i) = 12$

$\max(D_i) = 14$, $\max(E_i) = 15$ & $\min(F_i) = 17$

So, $\min(A_i) \geq$ max of B_i, C_i, D_i, E_i

& $\min(F_i) \geq$ max of B_i, C_i, D_i, E_i

Now, let G and H are two machines then processing time are as follows –

Jobs	machine ↓ G = A+B+C+D+E	H = B+C+D+E+F
1	49	60
2	58	53
3	60	58
4	61	60

The optimal sequence of jobs is as –

1	2	4	3

To calculate elapsed time, we use some notations –

Time in → Tin, Processing time → T_P,

Time out → Tout & Idle Time → T_i

Job Sequence	Machine A			machine B				machine C				machine D				machine E				machine F			
	Tin	Tp	Tout	Tin	Tp	Tout	Ti	Tin	Tp	Tout	Ti	Tin	Tp	Tout	Ti	Tin	Tp	Tout	Tc	Tin	Tp	Tout	Ti
1	0	15	15	15	8	23	15	23	6	29	23	29	14	43	29	43	6	49	43	49	26	75	49
2	15	17	32	32	7	39	9	39	9	48	10	48	10	58	5	58	15	73	9	75	22	97	–
4	32	18	50	50	6	56	11	56	11	67	8	67	12	79	9	79	14	93	6	97	17	114	–
3	50	21	71	71	7	78	15	78	12	90	11	90	9	99	11	99	11	110	6	114	19	133	–
							50				52				54				64				49

So, minimum elapsed time = 133 − 0 = 133

Idle time for machine A = 133 − 71 = 62 hrs

machine B = 133 − 78 + 50 = 105 hrs

machine C = 133 − 90 + 52 = 95 hrs

machine D = 133 − 99 + 54 = 88 hrs

machine E = 133 − 110 + 64 = 87 hrs

machine F = 49 hrs

Ques Solve the following problem—

Job	Machines (Processing time in hours)			
	M_1	M_2	M_3	M_4
A	12	5	3	12
B	9	2	9	9
C	13	3	4	13
D	14	3	3	14

Type - IV Processing two Jobs through m machines :-

Let there be two jobs A and B each of which is to be processed on m machines say $M_1, M_2, \ldots M_m$ in two different orders. (which is known in advance). The processing time on the given machines are also known and each machine can perform only one job at a time.

The objective is to determine an optimal sequence of processing the jobs so as to minimize the total elapsed time.

We follow the graphical procedure to find the minimum elapsed time.

Graphical procedure —

1) Draw two axes X and Y at right angles to each other. X axis representing processing times for job 1 which job 2 remain idle, Y axis representing the processing time for job 2 while job 1 remains idle.

2) Mark the processing times for job 1 and job 2 on X and Y axis respectively according to the given order of machines.

3) Construct the various blocks (rectangles) for various machines starting from the origin by pairing the same machines until the end point.

4) Draw the line starting from origin to end point by moving horizontally, vertically and diagonally along a line which makes an angle of 45° with the horizontal line (Base) till the point marked finished is reached.

The horizontal segment of this line indicates the first job is under process while second job is idle.

Similarly, the vertical segment of the line indicates that the second job is under process while first job is idle.

The diagonal segment of the line shows that both the jobs are under process simultaneously.

5) Find the optimal path, that minimizes the idle time for both the jobs.

So, choose the path which coincides with 45° line (diagonal movement) to the maximum extent.

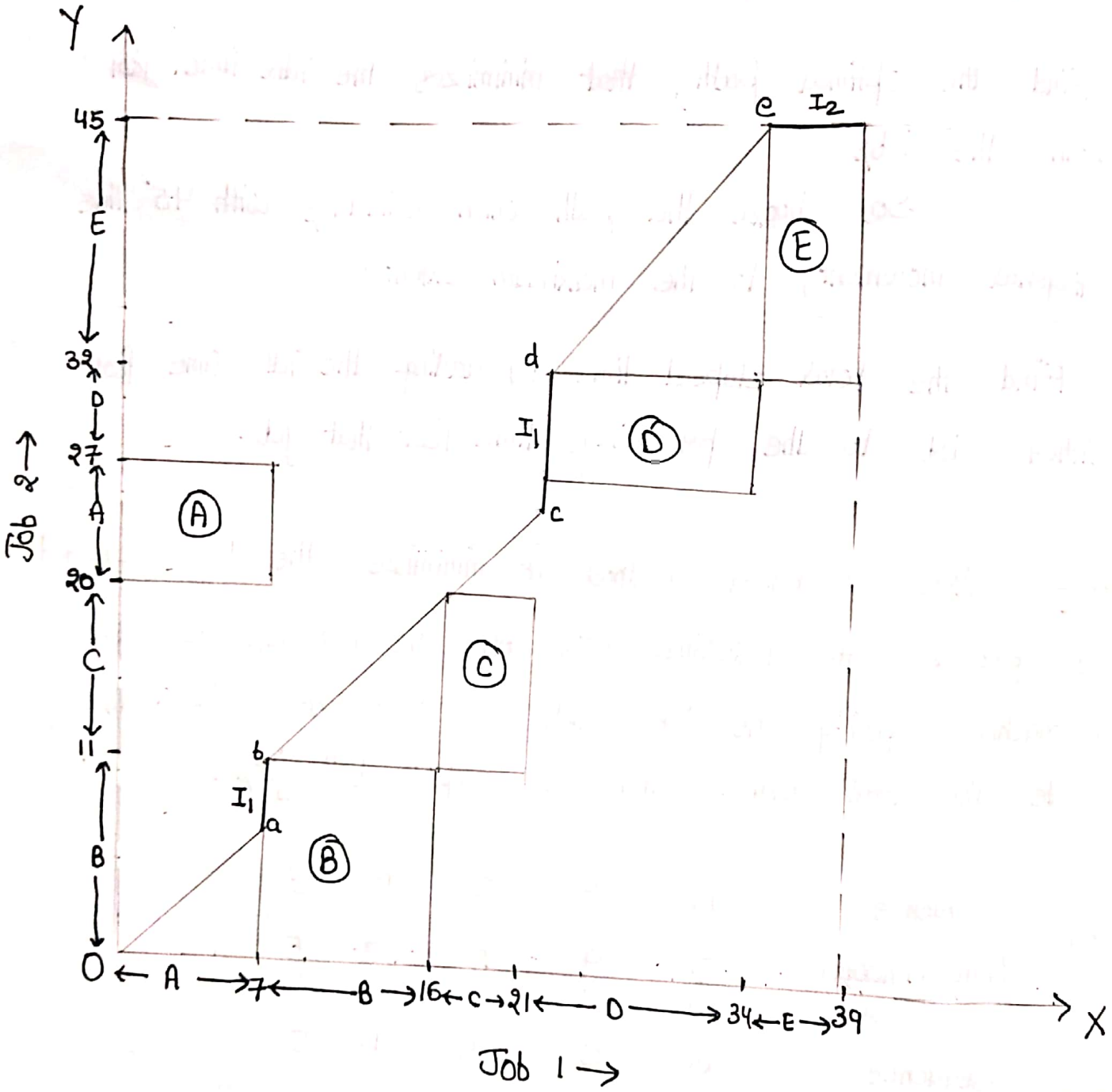
6) Find the total elapsed time by adding the idle time for either job to the processing time for that job.

Example- Use graphical method to minimize the time required to process the following jobs on the machines i.e., for each machine specify the job which should be done first. Also calculate the total elapsed time to complete both jobs.

Job 1	Sequence	A	B	C	D	E
	Time (in hours)	7	9	5	13	5
Job 2	sequence	B	C	A	D	E
	Time (in hours)	11	9	7	5	13

Sol Represent processing time on job 1 along horizontal axes (x-axis) and processing time on job 2 along vertical axes (y-axis).

Construct the graph as –



Optimal path – Oabcde

idle time for job 1 = 4+7 = 11 hours

idle time for job 2 = 5 hours

total elapsed time = 39+11 = 50 hours (job 1)

= 45+5 = 50 hours (job 2)

Example - Use the graphical method to find the minimum elapsed total time sequence of 2 jobs and 5 machines, when the following information is given —

		Machines				
	Sequence	A	B	C	D	E
Job 1	Time (in hours)	2	3	4	6	2
Job 2	Time (in hours)	C	A	D	E	B
		4	5	3	2	6