MGKV — Discrete Mathematics — BCA 2nd sem

Set Theory

Set — Set is well defined collection of elements, data or objects. It is denoted by A, B, C..... (capital letters)

Element of set is denoted by a, b, c... x, y, z (small letters)

Example – 1. set of natural number

i.e. $A = \{1, 2, 3, 4, \ldots\ldots\}$

2. set of vowels

$X = \{a, e, i, o, u\}$

Note – If the order of the elements is changed or any element of a set is repeated, it does not make any change in the set.

Representation of a set — sets can be represented in two ways –

1) <u>Roster or Tabular Form</u> — In this form, the elements of set are enclosed within bracket and separated by commas.

<u>Example</u> – (i) set of odd numbers less than 10.

$$A = \{1, 3, 5, 7, 9\}$$

2) <u>Rule Method or Set Builder Notation</u> — The set is defined by specifying a property that elements of the set have in common. The set is described as –

$$A = \{x : \text{property of } x\}$$

x : read as 'x such that'.

<u>Example</u> (i) $A = \{1, 3, 5, 7, 9\}$ is written as –

$$A = \{x : x < 10,\ x \text{ is odd natural no}\}$$

(ii) $X = \{a, e, i, o, u\}$ is defined as –

$$X = \{x : x \text{ is vowels}\}$$

Cardinality of Set – Number of elements of the set is called cardinality or cardinal number of set.

Let A be given set then cardinality of set is denoted by $n(A)$ or $|A|$.

Example – (i) $P = \{2, 4, 6, 8, 10\}$

then $n(P) = 5$

(ii) $X = \{1, 2, 3, 4, 5, \ldots\ldots\}$

then $n(X) = \infty$

Types of Sets – Sets can be classified into many types. Some of which are as follows –

1) **Null Set** – A set contains no elements is called null set or empty set. It is denoted by ϕ or $\{\}$.

Example – $A = \{x : x \in N \text{ and } 5 < x < 6\}$

is a null set i.e. $A = \phi$

2) **Singleton Set** – Singleton set or unit set contains only one element.

Example - $S = \{x : x \in N, 7 < x < 9\} = \{8\}$

3) **Finite Set and Infinite Set** –

If number of element of set is countable and finite then it is called finite set.

A set which is not finite or not countable is known as infinite set.

Example (i) $A = \{2, 4, 6, 8, 10\}$ finite set

(ii) $A = \{x : x \in N \text{ and } x > 100\}$ Infinite set

4) **Equal Sets** – Two sets are said to be equal, if they have exactly same elements (order does not matter)

Let A and B are two sets then it is denoted by $\boxed{A = B}$ equal set

Example $A = \{1, 2, 3\}$, $B = \{2, 3, 1, 1, 1\}$

then $\boxed{A = B}$

5) **Equivalent set** - Two sets A and B are said to be equivalent if they have equal no of elements.

i.e. $\boxed{n(A) = n(B)}$

Example - $A = \{2, 3, 4\}$, $B = \{a, b, c\}$

then $n(A) = n(B) = 3$

So, A and B are equivalent sets.

6) **Subset** — A set X is a subset of set Y, if every element of set X is an element of set Y. It is denoted by $X \subseteq Y$.

Example - $X = \{1, 2, 3\ 4\}$ and $Y = \{1, 2\}$

Here, set Y is a subset of set X.

Hence, $Y \subseteq X$

Note - Null set is a subset of each set.

Let set A has m elements then total number of subsets $= 2^m$

Example - (i) $A = \{1, 2\}$

total number of subset $= 2^2 = 4$

subsets are $= \phi, \{1\}, \{2\}, \{1, 2\}$

(ii) $A = \{a, b, c\}$

total number of subset $= 2^3 = 8$

subsets are $= \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\},$

$\{a, b, c\}$

(iii) $P = \{\phi, \{\phi\}\}$

total number of subset $= 2^2 = 4$

subsets are $= \phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}$

Super sets - If $A \subseteq B$ then B is called a super set of A and we write $B \supseteq A$

Proper Subset - If $A \subseteq B$ and $A \neq B$ then A is called a proper subset of B and we write $A \subset B$.

Example – Let $A = \{a, b, c\}$ & $B = \{a, b, c, d, e\}$

then every element of A is an element of B

And $A \neq B$.

Hence, 'A is proper subset of B' i.e. $A \subset B$.

Power set – The set of all subset of a given set 'A' is called power set of A. It is denoted by P(A).

Example –

$$A = \{1, 2, 3\}$$

$$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Comparable Sets – Two sets A and B are said to be comparable if one of them is subset of the other i.e. either $A \subseteq B$ or $B \subseteq A$.

Example the sets $\{1, 3, 4, 5\}$ and $\{1, 2, 3, 4, 5\}$ are comparable sets.

Universal Set – It is a collection of all elements of given set. Universal sets are represented as U.

Example– $A = \{1,2,3\}$, $B = \{4,5\}$, $C = \{5,6\}$

$U = \{1,2,3,4,5,6\}$

Operations of sets –

1) **Union –** Let A and B are two sets then union is the set of all those elements which are either in A or in B or in both A and B. It is denoted by '$A \cup B$'.

i.e. $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Example – $A = \{2,4,6,8\}$ & $B = \{3,6,9\}$

then $A \cup B = \{2,3,4,6,8,9\}$

2) **Intersection –** Intersection of two sets A and B is represented by '$A \cap B$'. It is the set of all those element which are common to both A and B.

So, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Example – $A = \{a, b, c, d, e\}$ and $B = \{a, e, i, o, u\}$

then $A \cap B = \{a, e\}$

3) Difference – The set difference of sets A and B is denoted by A–B is the set of elements which are only in A but not in B.

i.e. $A - B = \{x : x \in A \text{ and } x \notin B\}$

and $B - A = \{x : x \in B \text{ and } x \notin A\}$

Example – $A = \{1, 2, 3, 6\}$ and $B = \{1, 2, 4, 8\}$

then $A - B = \{3, 6\}$

and $B - A = \{4, 8\}$

4) Complement :– The complement of a set A is the set of elements which are not in set A. It is denoted by A'.

Hence, $A' = \{x : x \notin A\}$

i.e. $A' = U - A$

Example - If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

and $A = \{2, 4, 6, 8\}$

then $A' = U - A$

$= \{1, 3, 5, 7\}$

Disjoint set - Two set A and B are called disjoint if $\boxed{A \cap B = \phi}$

Ex

$A = \{2, 4, 6, 8\}$ & $B = \{1, 3, 5, 7\}$

then $A \cap B = \phi$.

So, A and B are disjoint set.

Intersecting Set - Two set A and B are said to be intersecting if $\boxed{A \cap B \neq \phi}$

Ex

$A = \{a, b, c\}$ & $B = \{a\}$

then $A \cap B = \{a\}$

Hence, they are intersecting sets.

Symmetric Difference – Let A and B are two sets then symmetric Difference is denoted by $A \triangle B$ and defined as –

$$\boxed{A \triangle B = (A-B) \cup (B-A)}$$

Example – Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{5, 4, 6, 7\}$

then
$$A - B = \{1, 2, 3\}$$
&
$$B - A = \{7, 6\}$$

So,
$$A \triangle B = \{1, 2, 3\} \cup \{7, 6\}$$
$$= \{1, 2, 3, 6, 7\}$$

Ex If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ & $C = \{3, 5, 7\}$

then
$$A \cap (B \cup C) = \{1, 2, 3\} \cap \{3, 4, 5, 7\}$$
$$= \{3\}$$

and
$$A - (B \cap C) = \{1, 2, 3\} - \{3\}$$
$$= \{1, 2\}$$

Some Important term for proving -

Let A and B are two sets -

1) $x \in (A \cup B) \Leftrightarrow x \in A$ or $x \in B$

2) $x \in (A \cap B) \Leftrightarrow x \in A$ and $x \in B$

3) $x \in (A - B) \Leftrightarrow x \in A$ and $x \notin B$

4) $x \in (B - A) \Leftrightarrow x \in B$ and $x \notin A$

5) $x \in A' \Leftrightarrow x \notin A$

6) $x \in A \Leftrightarrow x \notin A'$

7) $x \notin (A \cup B) \Leftrightarrow x \notin A$ and $x \notin B$

8) $x \notin (A \cap B) \Leftrightarrow x \notin A$ or $x \notin B$

9) $x \in P(A) \Leftrightarrow x \subseteq A$

10) $A = B \Rightarrow A \subseteq B$ and $B \subseteq A$

Algebra of Set Theory — There are some important laws of algebra which the set theory follows. These are as follows —

1) **Commutative Law** :— Let A and B are two sets then (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

Proof (i)

L.H.S., Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

$\therefore A \cup B \subseteq B \cup A$ ——— (I)

R.H.S.

Let $x \in B \cup A$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$

$\therefore B \cup A \subseteq A \cup B$ ——— (II)

from eq (i) & (II), we get

$\boxed{A \cup B = B \cup A}$ proved

(ii) L.H.S.

Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in B \cap A$

$\therefore A \cap B \subseteq B \cap A$ ——— (I)

R.H.S.

Let $x \in B \cap A$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$

$\therefore B \cap A \subseteq A \cap B$ ——— (II)

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from eq (i) & (ii), we have

$$\boxed{A \cap B = B \cap A}$$ proved

2) **Associative Law** – Let A, B and C are three sets then

(i) $A \cup (B \cup C) = (A \cup B) \cup C$

ii) $A \cap (B \cap C) = (A \cap B) \cap C$

Proof (i) L.H.S.

Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in (A \cup B) \cup C$

$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C$ —— (i)

R.H.S.

Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$

$\therefore (A \cup B) \cup C \subseteq A \cup (B \cup C)$ —— (ii)

from eq (i) & (ii), we have

$$\boxed{A \cup (B \cup C) = (A \cup B) \cup C}$$

(ii) L.H.S.

Let $x \in A \cap (B \cap C)$

$\Rightarrow x \in A$ and $x \in (B \cap C)$

$\Rightarrow x \in A$ and $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ and $x \in C$

$\Rightarrow x \in (A \cap B)$ and $x \in C$

$\Rightarrow x \in (A \cap B) \cap C$

$\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C$ ——— (I)

R.H.S.

Let $x \in (A \cap B) \cap C$

$\Rightarrow x \in (A \cap B)$ and $x \in C$

$\Rightarrow (x \in A$ and $x \in B)$ and $x \in C$

$\Rightarrow x \in A$ and $(x \in B$ and $x \in C)$

$\Rightarrow x \in A$ and $x \in (B \cap C)$

$\Rightarrow x \in A \cap (B \cap C)$

$\therefore (A \cap B) \cap C \subseteq A \cap (B \cap C)$ ——— (II)

from eq (i) & (ii), we get

$$\boxed{A \cap (B \cap C) = (A \cap B) \cap C}$$

proved

3) <u>Distributive Law</u> – Let A, B and C are three sets then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof (i) L.H.S.

Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore x \in A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ —— (1)

R.H.S.

Let $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A \cup (B \cap C)$

$\therefore x \in (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ —— (II)

from eq (I) & (II), we have

$$\boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$$ proved

(ii) L.H.S.

$x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ —— (i)

R.H.S.

$x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A \cap (B \cup C)$

$\therefore$ $\cancel{A \cap (B \cup C) \subseteq}$

$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ —— (ii)

from eq (i) & (ii), we have

$$\boxed{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$$ proved

4) **Demoorgan's Law** - Let A and B are two sets then

(i) $(A\cup B)' = A'\cap B'$ (ii) $(A\cap B)' = A'\cup B'$

Proof (i) L.H.S.

Let $x \in (A\cup B)'$

$\Rightarrow x \notin (A\cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A'\cap B'$

$\therefore (A\cup B)' \subseteq A'\cap B'$ —— (I)

R.H.S.

Let $x \in A'\cap B'$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin (A\cup B)$

$\Rightarrow x \in (A\cup B)'$

$\therefore A'\cap B' \subseteq (A\cup B)'$ —— (II)

Hence, $\boxed{(A\cup B)' = A'\cap B'}$ proved

(ii) L.H.S.

Let $x \in (A\cap B)'$

$\Rightarrow x \notin (A\cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A'\cup B'$

$\therefore (A\cap B)' \subseteq A'\cup B'$ —— (I)

R.H.S.

Let $x \in A'\cup B'$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin (A\cap B)$

$\Rightarrow x \in (A\cap B)'$

$\therefore A'\cup B' \subseteq (A\cap B)'$ —— (II)

So, $\boxed{(A\cap B)' = A'\cup B'}$ proved

5) Identity Law – Let A be any non-empty set then

(i) $A \cup \phi = A$ and (ii) $A \cap U = A$

where ϕ is null set & U is universal set.

Proof

(i) L.H.S.

Let, $x \in A \cup \phi$

$\Rightarrow x \in A$ or $x \in \phi$

$\Rightarrow x \in A$

$\therefore A \cup \phi \subseteq A$ ——— (I)

R.H.S.

Let, $x \in A$

$\Rightarrow x \in A$ or $x \in \phi$

$\Rightarrow x \in A \cup \phi$

$\therefore A \subseteq A \cup \phi$ ——— (II)

from eq (i) & (ii), we get

$\boxed{A \cup \phi = A}$ proved

(ii) L.H.S.

Let $x \in A \cap U$

$\Rightarrow x \in A$ and $x \in U$

$\Rightarrow x \in A$

$\therefore A \cap U \subseteq A$

R.H.S.

Let $x \in A$

$\Rightarrow x \in A$ and $x \in U$

$\Rightarrow x \in A \cap U$

$\therefore A \subseteq A \cap U$

from eq (i) & (ii), we get

$\boxed{A \cap U = A}$ proved

6) Idempotent Law – Let A be any set then

(i) $A \cup A = A$ (ii) $A \cap A = A$

Proof (i) L.H.S.

Let $x \in A \cup A$

$\Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A$

$\therefore A \cup A \subseteq A$ —— (I)

R.H.S.

Let $x \in A$

$\Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A \cup A$

$\therefore A \subseteq A \cup A$ —— (II)

So, $\boxed{A \cup A = A}$ proved

(ii) L.H.S.

Let $x \in A \cap A$

$\Rightarrow x \in A$ and $x \in A$

$\Rightarrow x \in A$

$\therefore A \cap A \subseteq A$ —— (I)

R.H.S.

Let $x \in A$

$\Rightarrow x \in A$ and $x \in A$

$\Rightarrow x \in (A \cap A)$

$\therefore A \subseteq A \cap A$ —— (II)

from eqn (I) & (II), we have

$\boxed{A \cap A = A}$ proved

Ques - Prove that -

(i) $A-(B\cup C) = (A-B)\cap(A-C)$

(ii) $A-(B\cap C) = (A-B)\cup(A-C)$

Proof - (i) L.H.S.

Let, $x \in A-(B\cup C)$

$\Rightarrow x \in A$ and $x \notin (B\cup C)$

$\Rightarrow x\in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow (x\in A$ and $x\notin B)$ and $(x\in A$ and $x\notin C)$

$\Rightarrow x\in(A-B)$ and $x\in(A-C)$

$\Rightarrow x\in (A-B)\cap(A-C)$

$\therefore$ $A-(B\cup C) \subseteq (A-B)\cap(A-C)$ —— (I)

R.H.S. Let $x\in(A-B)\cap(A-C)$

$\Rightarrow x\in(A-B)$ and $x\in(A-C)$

$\Rightarrow (x\in A$ and $x\notin B)$ and $(x\in A$ and $x\notin C)$

$\Rightarrow x\in A$ and $(x\notin B$ and $x\notin C)$

$\Rightarrow x\in A$ and $x\notin(B\cup C)$

$\Rightarrow x\in A-(B\cup C)$

$\therefore$ $(A-B)\cap(A-C) \subseteq A-(B\cup C)$ —— (II) from eq^n (i) & (ii), we get

$$\boxed{A-(B\cup C) = (A-B)\cap(A-C)}$$ proved

(ii) Same as (i).

Ques – For any two set A and B, prove that

$$P(A\cap B) = P(A)\cap P(B)$$

Proof

L.H.S. Let, $x \in P(A\cap B)$

$\Rightarrow x \subseteq A\cap B$

$\Rightarrow x \subseteq A$ and $x \subseteq B$

$\Rightarrow x \in P(A)$ and $x \in P(B)$

$\Rightarrow x \in P(A)\cap P(B)$

$\therefore P(A\cap B) \subseteq P(A)\cap P(B)$ —— (i)

R.H.S.

Let, $x \in P(A)\cap P(B)$

$\Rightarrow x \in P(A)$ and $x \in P(B)$

$\Rightarrow x \subseteq A$ and $x \subseteq B$

$\Rightarrow x \subseteq A\cap B$

$\Rightarrow x \in P(A\cap B)$

$\therefore P(A)\cap P(B) \subseteq P(A\cap B)$ —— (ii)

So,

$$\boxed{P(A\cap B) = P(A)\cap P(B)}$$ proved

Cartesian Product :— Let A and B are two non-empty sets then cartesian product of A and B is denoted by '$A \times B$' and defined as —

$$A \times B = \{ (x, y) : x \in A \text{ and } y \in B \}$$

i.e., it is set of ordered pair element, where first element is belongs to first set and second element of ordered pair is belongs to 2nd set.

Example — If $A = \{1, 2, 3\}$ & $B = \{a, b\}$ then

$$A \times B = \{ (1,a), (1,b), (2,a), (2,b), (3,a), (3,b) \}$$

& $$B \times A = \{ (a,1), (a,2), (a,3), (b,1), (b,2), (b,3) \}$$

Ques — Let $A = \{1, 2\}$, $B = \{2, 3\}$ & $C = \{2, 4\}$ then verify that —

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

& (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol Now,

$$(B \cup C) = \{2, 3, 4\}$$

$$B \cap C = \{2\}$$

$$A \times B = \{ (1,2), (1,3), (2,2), (2,3) \}$$

& $$A \times C = \{ (1,2), (1,4), (2,2), (2,4) \}$$

So, (i) L.H.S.

$A \times (B \cup C) = \{1,2\} \times \{2,3,4\}$

$= \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4)\}$

R.H.S. $(A \times B) \cup (A \times C)$

$= \{(1,2), (1,3), (2,2), (2,3)\} \cup \{(1,2), (1,4), (2,2), (2,4)\}$

$= \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4)\}$

Hence, L.H.S = R.H.S. verified

(ii) L.H.S. $A \times (B \cap C)$

$= \{1,2\} \times \{2\}$

$= \{(1,2), (2,2)\}$

R.H.S. $(A \times B) \cap (A \times C)$

$= \{(1,2), (1,3), (2,2), (2,3)\} \cap \{(1,2), (1,4), (2,2), (2,4)\}$

$= \{(1,2), (2,2)\}$

So, L.H.S. = R.H.S. Verified

Ques – For three non empty sets A, B & C prove that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Proof

(i) L.H.S. Let $(x,y) \in (A \times (B \cup C))$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$

$\Rightarrow (x,y) \in (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ ——— (I)

R.H.S. Let $(x,y) \in (A \times B) \cup (A \times C)$

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$

$\Rightarrow (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow (x,y) \in A \times (B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ ——— (II) from eq (i) & (ii), we have

$$\boxed{A\times(B\cup C) = (A\times B)\cup(A\times C)}$$ proved

(ii) L.H.S. Let $(x,y) \in A\times(B\cap C)$

$\Rightarrow x\in A$ and $y\in(B\cap C)$

$\Rightarrow x\in A$ and $(y\in B$ and $y\in C)$

$\Rightarrow (x\in A$ and $y\in B)$ and $(x\in A$ and $y\in C)$

$\Rightarrow (x,y)\in(A\times B)$ and $(x,y)\in(A\times C)$

$\Rightarrow (x,y)\in(A\times B)\cap(A\times C)$

$\therefore A\times(B\cap C) \subseteq (A\times B)\cap(A\times C)$ ——— (i)

R.H.S. $(x,y)\in(A\times B)\cap(A\times C)$

$\Rightarrow (x,y)\in(A\times B)$ and $(x,y)\in(A\times C)$

$\Rightarrow (x\in A$ and $y\in B)$ and $(x\in A$ and $y\in C)$

$\Rightarrow x\in A$ and $(y\in B$ and $y\in C)$

$\Rightarrow x\in A$ and $y\in(B\cap C)$

$\Rightarrow (x,y)\in A\times(B\cap C)$

$\therefore (A\times B)\cap(A\times C) \subseteq A\times(B\cap C)$ ——— (ii)

from eqn (i) & (ii), we get

$$\boxed{A\times(B\cap C) = (A\times B)\cap(A\times C)}$$ proved

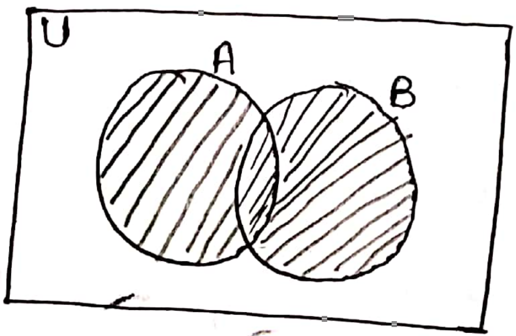
Venn Diagram – A venn diagram is a diagram that shows the relationship among sets, we represent them pictorially by means of diagram.

In these diagrams, the universal set is represented by a rectangle region and its subsets (i.e. all the sets) by circles inside the rectangles.

The various operations of set theory are represented as –

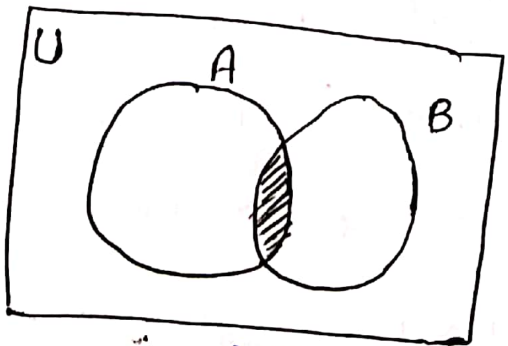
1) **Union** –

The union of two sets A and B is represented by shaded portion of the venn diagram.



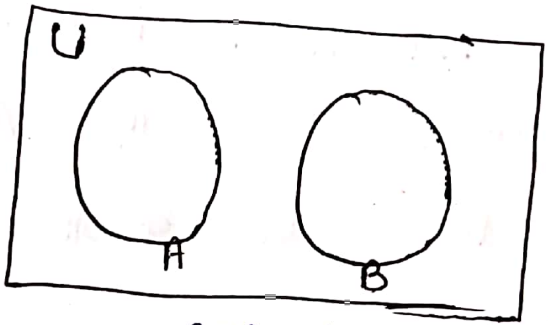
A∪B

2) **Intersection** –



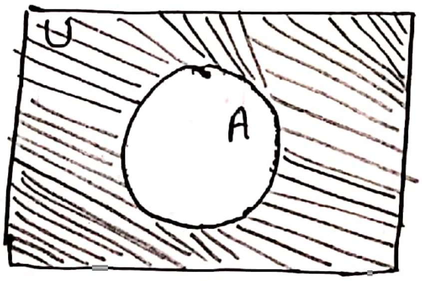
A∩B.

3) Disjoint set – Two sets are called disjoint if they have no common values.



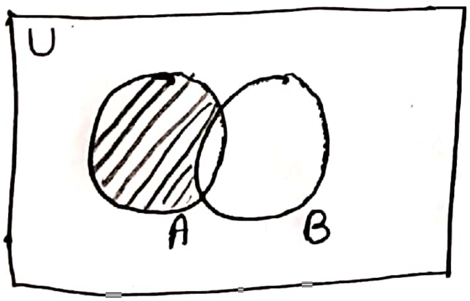
$A \cap B = \phi$

4) Complement –

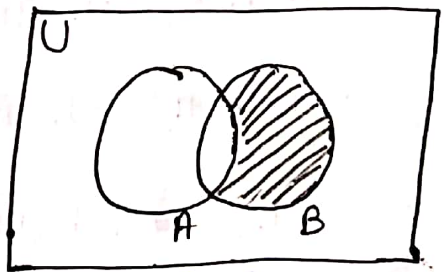


$A' = U - A$

5) Difference –



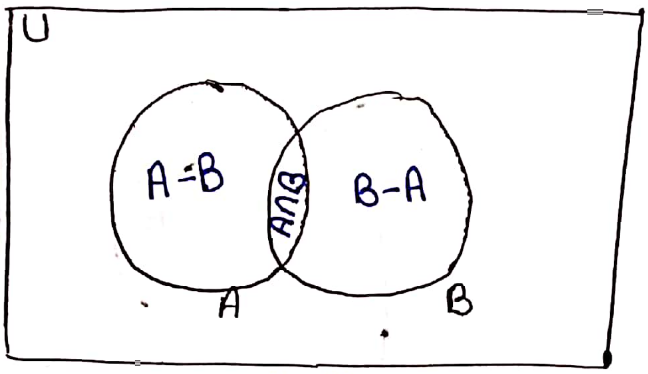
$A - B$



$B - A$

Formula By using Venn Diagram –

Let A and B are two intersecting subsets of universal set U then venn diagram of operations of set theory are shown as–



$A \cup B$

From the Venn diagram, it is clear that –

1) $n(A) = n(A-B) + n(A \cap B)$

2) $n(B) = n(B-A) + n(A \cap B)$

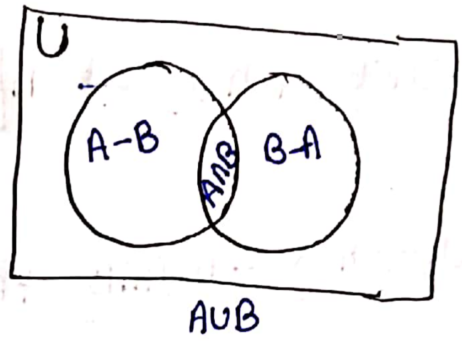
3) $n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A)$

Theorem – For any set A and B, prove that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Proof -

we have by venn diagram



$n(A \cup B) = n(A-B) + n(A \cap B)$
$+ n(B-A)$

Now, add & subtract $n(A \cap B)$ in R.H.S., we get

$$n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A) + n(A \cap B) - n(A \cap B) \quad \text{---(I)}$$

From the venn diagram, it is also clear that

$$\left.\begin{aligned} n(A) &= n(A-B) + n(A \cap B) \\ \& \quad n(B) &= n(B-A) + n(A \cap B) \end{aligned}\right\} \quad \text{---(II)}$$

from eq (i) & (II), we get

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$ proved

Theorem - For any three set A, B, C prove that

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

Proof

L.H.S.

$n[(A\cup B)\cup C]$

$= n(A\cup B) + n(C) - n[(A\cup B)\cap C]$ $(\because n(A\cup B) = n(A) + n(B) - n(A\cap B))$

$= n(A) + n(B) - n(A\cap B) + n(C) - n[(A\cap C)\cup(B\cap C)]$

(using distributive law)

$= n(A) + n(B) + n(C) - n(A\cap B) - [n(A\cap C) + n(B\cap C) - n[(A\cap C)\cap(B\cap C)]]$

$= n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$ R.H.S.

proved.

Ques In a group of 60 people, 40 speak ~~Hindi~~ English, 20 speak both English & Hindi and all people speak at least one of the two languages. How many speak only English and not hindi? How many speak ~~English?~~ Hindi?

Sol - Let H denotes hindi and E represent english.

Given that -

$$n(H \cup E) = 60$$

$$n(E) = 40$$

$$n(H \cap E) = 20$$

then, number of people that speak only English and not Hindi

$$n(E - H) = n(E) - n(E \cap H)$$

$$= 40 - 20 = 20 \quad \underline{\text{Ans}}$$

Since, we know that

$$n(H \cup E) = n(H) + n(E) - n(H \cap E)$$

$$60 = n(H) + 40 - 20$$

$$n(H) = 60 - 20 = 40 \text{ (speak hindi)} \quad \underline{\text{Ans}}$$

Ques In a school of 1000 students, some students play hockey and some play basket ball and others do not play any game. There is a total of 400 students who play hockey, 450 play basketball and 50 play both the game. Find out

(a) No of student who play only hockey.

(b) No of student who play only basket ball.

(c) who do not play any game.

Sol

Let H represent hockey and B represent basketball. Given that

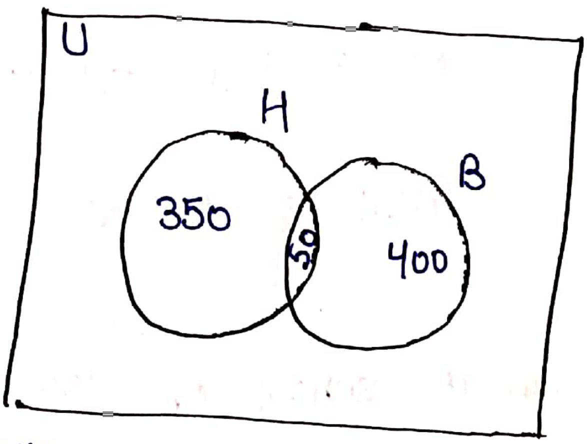
$n(U) = 1000$ (total student)

$n(H) = 400$

$n(B) = 450$

$n(H \cap B) = 50$

using venn diagram



Students do not play any game

= Total students − No of students who play any one of game

= 1000 − (350 + 50 + 400)

= 200 students

only hockey = 350

only Basket ball = 400

Using Formula –

(a) No of student who play only hockey

$$n(H-B) = n(H) - n(H \cap B)$$

$$= 400 - 50 = 350 \text{ student}$$ Ans

(b) No of student who play only basketball.

$$n(B-H) = n(B) - n(H \cap B)$$

$$= 450 - 50 = 400 \text{ student}$$ Ans

(c) we know that

$$n(B \cup H) = n(B) + n(H) - n(B \cap H)$$

$$= 450 + 400 - 50$$

$$= 800$$

So, No of students do not play any game

$$n(B \cup H)' = n(U) - n(B \cup H)$$

$$= 1000 - 800$$

$$= 200 \text{ student}$$ Ans

Ques A School has 21 boys in basketball team, 26 in hockey and 29 in football team. Now if 14 boys play hockey and basketball, 15 boys play hockey and football, 12 boys play football and basketball and 8 boys play hockey, football and basketball all three games, then what is total number of boys playing these games?

Sol Let B, H, F denote the total number of boys playing basketball, hockey and football.

then $n(B) = 21$, $n(H) = 26$, $n(F) = 29$

$n(H \cap B) = 14$, $n(H \cap F) = 15$, $n(F \cap B) = 12$

and $n(B \cap H \cap F) = 8$

then,

$$n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$$

$$= 21 + 26 - 29 - 14 - 15 - 12 + 8$$

$$= 84 - 41$$

$$= 43 \text{ student}$$ **Ans**

Ques- In a group of 52 persons, 16 drink tea but not coeffe and 33 drink tea.

i) How many drink tea and coffee both?

ii) How many drink coffee but not tea?

Sol

Let, T = set of person who drink tea

C = set of person who drink coffee

$n(T \cup C) = 52, \quad n(T) = 33, \quad n(T-C) = 16$

(i) set of person who drink tea and coffee both

$n(T) = n(T-C) + n(T \cap C)$

or $33 = 16 + n(T \cap C)$

or $n(T \cap C) = 33 - 16 = 17$ Ans

(ii) set of person who drink coffee but not tea is calculated as –

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

or $52 = 33 + n(C) - 17$

or $n(C) = 52 - 16 = 36$

Now, $n(C-T) = n(C) - n(C \cap T)$

$= 36 - 17 = 19$ Ans